

CONTENTS

Introduction	1
Chapter I. Preliminaries on non-crossing partitions	3
1.1. Non-crossing partitions	3
1.2. Incidence algebra and convolution	6
1.3. Multiplicative functions	7
Chapter II. Operator-valued multiplicative functions on the lattice of non-crossing partitions	11
2.1. Operator-valued multiplicative functions	11
2.2. Connection between \hat{f} and $\hat{f} * \zeta$	15
2.3. Special case $\mathbf{I} = \mathbf{I}(\mathbf{C}, \mathbf{C})$	17
2.4. Tracial multiplicative functions	22
2.5. Product and cluster property	23
Chapter III. Amalgamated free products	27
3.1. Basic notations	27
3.2. Moment and cumulant functions	28
3.3. Definition of the amalgamated free product	32
3.4. Explicit formula for $\varphi_1 * \varphi_2$	36
3.5. Positivity of the amalgamated free product	43
Chapter IV. Operator-valued free probability theory	47
4.1. B -valued random variables and free convolution	47
4.2. B -Gaussian distributions and central limit theorem	54
4.3. Positivity of B -Gaussian distributions	56
4.4. Compound B -Poisson distributions	60
4.5. Infinitely divisible distributions	62
4.6. Full Fock space over a Hilbert- B -bimodule	64
4.7. Realization of infinitely divisible distributions on a full Fock space	71
Chapter V. Operator-valued stochastic processes and stochastic differential equations	75
5.1. B -valued stochastic processes	75
5.2. Formulation of the problem	78
5.3. Possible solutions of the problem	79
5.4. Gaussian approximation	84
Bibliography	87